Activity - Modeling the Spread of an Infectious Disease

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Introduction: Imagine that you are one of many people at a party and that, unknown to everyone else, one person arrives carrying an infectious disease. How quickly will the disease spread, and what are the chances that you will leave the party with the disease?

Data Collection Chart: Your ID Number: 24

Let N= the number of students in class, and let A(0)=1. 27

math → prb → #5 → randInt(1,27) → rand enter

seed 1234 → rand

Data #1

Stage Number	Number of Newly Infected Individuals	Number of Total Infected Individuals
0	—	1
1	1	2
2	2	4
3	3	7
4	5	12
5	5	17
6	5	22
7	4	26

Let N=60, and let A(0)=3.

randInt(1,27,#) → L1 : SortA(L1) : L1 enter

Data #2

Stage Number	Number of Newly Infected Individuals	Number of Total Infected Individuals
0	——	3
1	2	5
2	4	9
3	8	17
4	7	24
5	11	35
6	11	46
7	10	56
8	3	59
9	0	59
10	1	60

Analyzing the Data:

Use the data from the first chart.

1. Make a scatterplot of the "Stage Number" (in L1) vs. the "Number of Total Infected Individuals" (in L2).

2. Since the data (should) appear to be a model for a logistic function, we need to find a function in the form:

$$I(t) = \frac{c}{1+a\cdot e^{-bt}}$$

where curvature changes
try inflection point

Therefore, we need to find values for the three constants a, b, and c. The value of c should be easy. For our activity,

c= 27

if doesn't fit too well, try other points ex point #5 + the first one

To find a and b, select two other points from the table/scatterplot. (One should be the initial point, and the other should (possibly) be the next to the last point. Substitute the ordered pairs into the equation and solve for a and b. Show your work. (0, 1) (7, 26)

$1 = \frac{27}{1+ae^{-b(0)}}$ $\quad 1 = \frac{27}{1+a}$

$1 = \frac{27}{1+a}$ $\quad 1+a = 27$ $\quad a = 26$

$26 = \frac{27}{1+26e^{-7b}}$

$1+26e^{-7b} = \frac{27}{26}$

$e^{-7b} = .00148$

$\frac{-7b}{-7} = \frac{-6.5}{-7}$

$b = .931$

The final equation is: $I(t) = \frac{27}{1+26e^{-.931t}}$ Of course, graph it to see how it fits the scatterplot.

3. Find a logistic regression equation, and see if it fits the data. Do all of the values in the equation make sense to the data we collected?

$y = \frac{29.48}{1+29.42e^{-.747t}}$

No, because there is 29.48 and we only have 27 people in class

4. Put the "Number of Total Infected Individuals" from the second data chart in L3 and repeat the above steps to find the logistic function that models the data in L1 vs. L3.

$c = 60$ $\quad (0, 3)$ $\quad (7, 56)$

$y = \frac{60}{1+19e^{-.795t}}$ $\quad y = \frac{62.51}{1+26e^{-.798t}}$

$3 = \frac{60}{1+ae^{-b(0)}}$

$1+a = 20$

$a = 19$

$56 = \frac{60}{1+19e^{-b(7)}}$

$1+19e^{-7b} = \frac{60}{56}$

$\frac{19e^{-7b}}{19} \approx \frac{.0714}{19}$

$e^{-7b} \approx .00376$

$\frac{-7b}{-7} \approx \frac{-5.583}{-7}$

$b \approx .798$

5. How do the two models compare? How are they the same and how are they different?

Our model doesn't hit the middle points but hits the first and last, while the calculator goes through most points. Also the calculator also goes up to 62.5 and we only have 60 people.

Worksheet - Review of Geometric Transformations

In an algebra course, a topic that is emphasized is that of graphing basic functions, and applying geometric transformations to these basic functions to obtain the graph of a more "complex" function. The geometric tansformations that we study are:

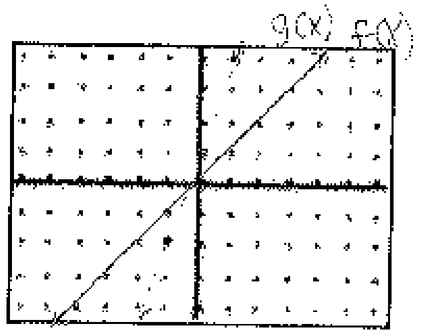
1) Vertical shift (up or down).
2) Horizontal shift (left or right).
3) Vertical stretch/shrink (by a factor).
4) Reflection through the x-axis.

In the following problems, a basic function f is listed followed by a more "complex" function g. List the geometric transformations used to obtain the graph of g from the basic function f. Then sketch both functions f and g.
Also, after graphing the two functions, for the basic function f(x), state the domain and range, and state intervals where function is increasing/decreasing.

1. $f(x)=x$, $g(x)=3x+1$

Transformations applied to g(x):

Vertical stretch by 3
Vertical shift up 1

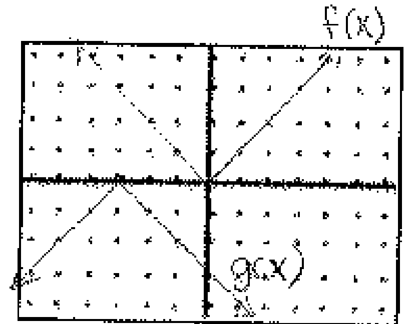


D_f: $(-\infty,\infty)$ R_f: $(-\infty,\infty)$
Inc: $(-\infty,\infty)$ Dec: —

2. $f(x)=|x|$, $g(x)=-|x+3|$

Transformations applied to g(x):

horizontal shift left 3
reflection through x-axis

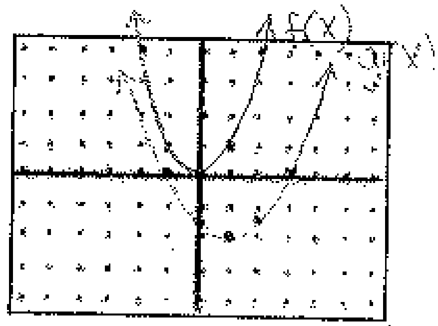


D_f: $(-\infty,\infty)$ R_f: $[0,\infty)$
Inc: $(0,\infty)$ Dec: $(-\infty,0)$

3. $f(x)=x^2$, $g(x)=\frac{1}{2}(x-1)^2-2$

Transformations applied to g(x):

horizontal shift right
vertical shrink by 1/2
vertical shift down 2

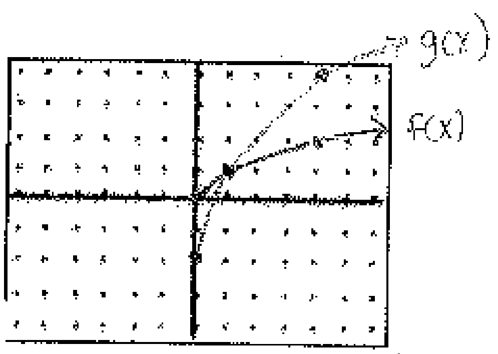


D_f: $(-\infty,\infty)$ R_f: $[0,\infty)$
Inc: $(0,\infty)$ Dec: $(-\infty,0)$

4. $f(x)=\sqrt{x}$, $g(x)=3\sqrt{x}-2$

Transformations applied to g(x):

Vertical stretch by 3
Vertical shift down 2

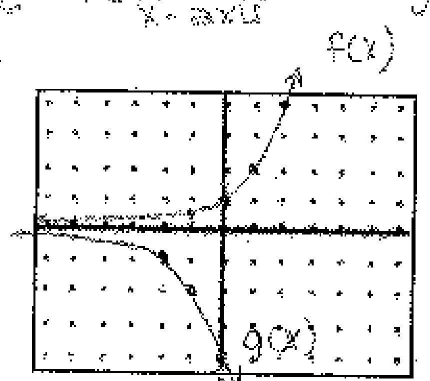


D_f: $[0,\infty)$ R_f: $[0,\infty)$
Inc: $(0,\infty)$ Dec: —

5. $f(x)=2^x$, $g(x)=-2^{x+2}$

Transformations applied to g(x):

horizontal shift left 2
reflection through x-axis

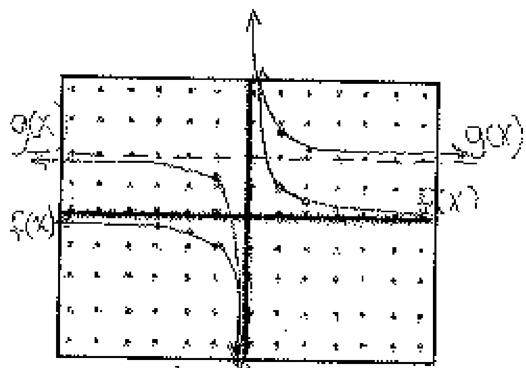


D_f: $(-\infty,\infty)$ R_f: $(0,\infty)$
Inc: $(-\infty,\infty)$ Dec: —

6. $f(x)=\frac{1}{x}$, $g(x)=\frac{2}{3x}+2$

Transformations applied to g(x):

vertical shrink by 2/3
vertical shift up 2



D_f: $(-\infty,0)(0,\infty)$ R_f: $(-\infty,0)(0,\infty)$
Inc: — Dec: $(-\infty,0)(0,\infty)$

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Logistic Equation: $y = \frac{c}{1+ae^{-kt}}$

WS Logistic - disease

#4 (0, 3) (6, 46)

$a = 19$

$46 = \frac{60}{1+19e^{-b(6)}}$

$1+19e^{-6b} = \frac{60}{46}$

$\frac{19e^{-6b}}{19} \approx \frac{.304}{19}$

$\ln e^{-6b} \approx \ln .016$

$\frac{-6b}{-6} \approx \frac{-4.136}{-6}$

$y = \frac{60}{1+19e^{-.689t}}$

$b \approx .689$

5. The second one with more people increases steeper, because of c value which is the stretch/shrink value. The b value which is the rate of people getting infected.

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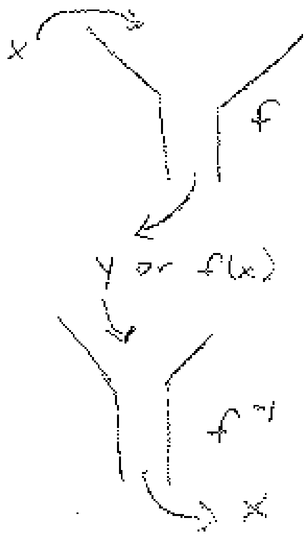
$y = a \cdot b^x$ or $n(t) = n_0 e^{kt}$

$k > 0$ increasing function

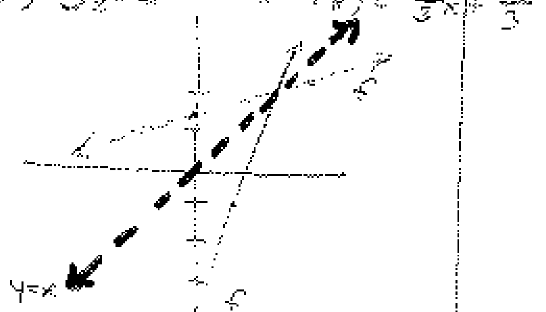
$k < 0$ decreasing function

$b = e^k$

$e^{-1/2} = \frac{1}{\sqrt{e}} = \sqrt{\frac{1}{e}}$



f^{-1} Inverse (Undoes the function)

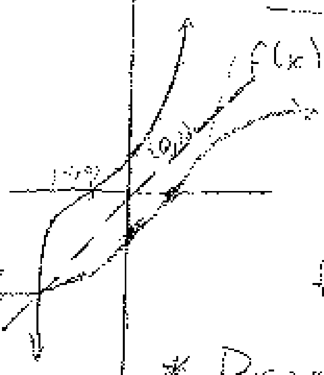


(*) property: f and f^{-1} are symmetric with respect to $y=x$

f	f^{-1}
$f(x) = x+2$	$f^{-1}(x) = x-2$
$f(x) = x/5$	$f^{-1}(x) = 5 \cdot x$
$f(x) = 3x-4$	$f^{-1}(x) = \frac{x+4}{3} = \frac{1}{3}x + \frac{4}{3}$
$f(x) = (x+1)^3$	$f^{-1}(x) = \sqrt[3]{x} - 1$

$f(7) = 17$ and $f^{-1}(17) =$

$f^{-1}(7) = \frac{11}{3}$ and $f(11/3) =$



(*) property: if (a,b) is on f then (b,a) is on f^{-1}

$f^{-1}(f(7)) = 7$ $\quad f(f^{-1}(7)) = 7$

* Property: $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$

(Turn graph over rotate) turn rotate